Self-organized growth model for a driven interface in random media

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We introduce a simple self-organized growth model in the quenched Edwards-Wilkinson universality class. The roughness and growth exponents are obtained for the model as $\alpha \simeq 1.15$ and $\beta \simeq 0.89$, respectively. These values of exponents are in good agreement with previous results obtained from numerical integrations or discretized model simulations of the quenched Edwards-Wilkinson equation. The velocity of a driven interface is found to be independent of the slope of the tilted substrate. [S1063-651X(99)13005-8]

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Recently the motion of driven interfaces in random media has attracted a great deal of interest because it is related to many physical phenomena such as fluid invasion in porous media [1,2], depinning of charge density waves [3], fluid imbibition in paper [4], driven flux motion in type-II superconductors [5,6], etc. A well known physical feature of the driven motion of an interface is an interplay between the quenched disorder and the driving force acting on the interface. The interface is pinned when the driving force F is smaller than the pinning strength induced by the quenched disorder. For a large F, however, the interface can move for a while until it is pinned again. There exists a threshold of the driving force F_c above which the interface moves with a certain velocity. Accordingly the velocity is zero for $F < F_c$, and it increases for $F > F_c$. This phenomenon is called the pinning-depinning transition.

The dynamics of driven interfaces in a random medium near the threshold ($F \to F_c$) has been well explained by the quenched Kardar-Parisi-Zhang (QKPZ) equation [7],

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h + \lambda (\nabla h)^2 + F + \eta(x,h), \tag{1}$$

where $h(x,t)$ is the height of the interface at position x and time t. F is an external driving force and η is a quenched noise with $\langle \eta(x,h) \rangle = 0$ and $\langle \eta(x,h)\eta(x',h') \rangle = 2D\delta(x-x')\delta(h-h')$. The global interface width, defined by $W(L,t) = \langle L^{-d'} \Sigma_x [h(x,t) - \overline{h}(t)]^2 \rangle^{1/2}$, is expected to scale as

$$W(L,t) \sim \begin{cases} t^{\beta} & \text{if} \quad t \ll L^z \\ L^{\alpha} & \text{if} \quad t \gg L^z. \end{cases} \tag{2}$$

Here $\overline{h}$, L, and d' denote the mean height, system size, and substrate dimension, respectively. The angular bracket stands for statistical average. α (β) is the roughness (the growth) exponent and $z = \alpha/\beta$ is the dynamic exponent. The most commonly measured exponent is the roughness exponent α.

A number of stochastic discrete models have been introduced to mimic the QKPZ equation, Eq. (1) [4,8]. A simple but notable growth model among them has been proposed by Sneppen several years ago [9]. The scaling properties of the surface of the model are found to be controlled by the directed percolation structure on the quenched random forces [10]. The obtained roughness exponent $\alpha \simeq 0.63$ gives an excellent agreement with a mapping to the directed percolation cluster and also the value obtained from direct numerical integration or discretized model simulation of the QKPZ equation.

Even though $\alpha \simeq 0.63$ for the QKPZ universality class is generally accepted, many experiments having rather larger values than 0.63 [1,11] lead to questions for other universality classes. The simplest possible choice is the quenched Edwards-Wilkinson (QEW) universality class, described by the EW equation with the quenched disorder [2],

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h + F + \eta(x,h). \tag{3}$$

It turns out that this linear equation has nontrivial scaling properties due to the quenched disorder. Many analytic and numerical works have been carried out to describe and understand the driven motion of interfaces following the QEW equation, Eq. (3). Analytical [12] and numerical (using the height-height correlation function) studies [13] give us a roughness exponent $\alpha \simeq 1$ for the surface dimension $d = 1$. However, an anomalous roughness exponent $\alpha > 1$ [14–16] has been found when the global scaling from the surface width is used. All the direct integrations of the QEW equation [17] and various models [14–16,18] corresponding to the discretization of this equation give $\alpha > 1$. There thus remain disagreements between many of the numerical and analytical results.

In spite of disagreements, it is generally accepted that the QEW universality class exists and a simple growth model could be found to describe the super-rough interface characterized by $\alpha > 1$. The Sneppen model is a self-organized growth model in the QKPZ universality class. An interesting feature of this model is that the growing surface is not controlled by an external driving force F but rather by the self-organized growth. Therefore it would be interesting if we can find a simple QEW model as an analogy of the Sneppen model in the QKPZ universality class. In this paper, we will introduce a simple discrete growth model belonging to the QEW universality class. Our model is not the discretized version of the QEW equation. The motion of the interface is not controlled by an external driving force, but rather by the quenched disorder on the surface and also by the self-organized growth.

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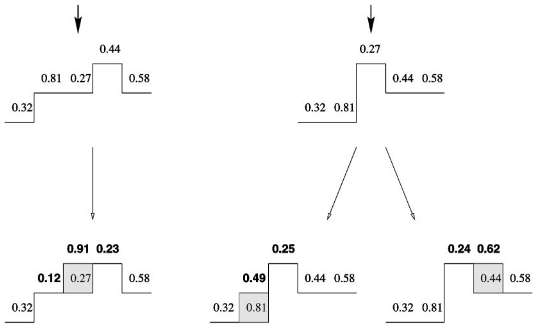

FIG. 1. Schematic representations of the stochastic rule of our model. The arrows indicate the selected sites having the lowest minimum random number. The gray squares denote the newly added particles on the interface. The bold numbers denote the newly updated random numbers.

The stochastic rule of our model in 1+1 dimensions is defined as follows: We preassign random numbers, representing impurities in a random medium, to all perimeter sites of the initially flat interface. At each time step, we add a particle on the selected site, which has a global minimum random number, and then an existing random number on that site is updated. After that we allow the newly added particle to relax randomly to one of the nearest neighbor sites whose height is lower than that of the selected site. We also update the random number at the newly occupied site. When there is no relaxation of an added particle, we update random numbers of two nearest neighbors as well as that of the selected site. The stochastic growing rule of our model is depicted in Fig. 1. In this way we can generate a downhill movement which is a universal feature appearing in all EW-type models. In the Sneppen model, there are avalanches of growth due to the restricted solid-on-solid (RSOS) condition. The RSOS condition actually results in a constraint to the slope [16] and generates the KPZ-type nonlinearity in Eq. (1). In our stochastic growth rule, we have only one avalanche to the site with lower height, no matter what the height difference between two sites. Therefore no constraint to the slope is imposed, indicating the absence of the KPZ nonlinearity. Moreover we update the existing random numbers of two nearest neighbors if the height of a selected site is not larger than those of two nearest neighbors. This process is also helpful to avoid any possible local slope dependence of the velocity of a growing interface. Hence we expect the generic QEW behavior in this model.

We have carried out Monte Carlo simulations for system sizes $L=64$, 128, 256, 512, and 1024. Numerical data are averaged typically over 100 configurations. In order to obtain the growth exponent for our model, we measure the time-dependent behavior of the global interface width $W(L,t)$ starting from an initially flat interface. We plot $W^2(L,t)$ versus time in double logarithmic scale in Fig. 2. The interface width initially grows with the growth exponent 0.5 as random growth. After that, the interface width grows with the exponent $\beta \simeq 0.89$ as shown in Fig. 2. This value of growth exponent is consistent with the direct integration result of the QEW equation [17] and the result from the automaton models in the QEW universality class [14–16,18]. We also consider another growth exponent by measuring the global

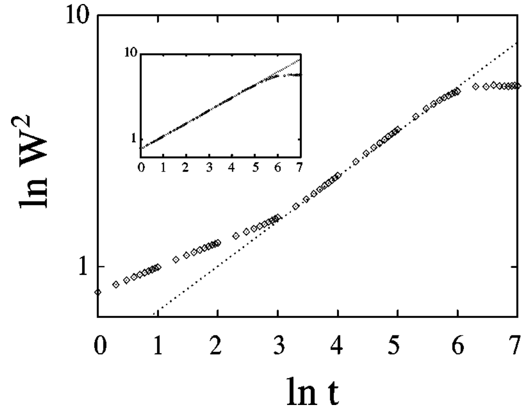

FIG. 2. Plot of $W^2(L,t)$ vs time t in double logarithmic scales for the system size $L=1024$. The slope of the dotted line is $\beta = 1.78$. Inset: The slope of the line is the same plot of $W^2(L,t)$ obtained with $2\beta_s = 1.5$.

width starting from the saturated interface instead of the flat interface. The growth exponent is measured as $\beta_s \simeq 0.75$. This value is also consistent with the one obtained from the QEW equation after saturation and analytic solution of the QEW equation [12,18]. The exponent β_s is smaller than β obtained from a flat interface. In the Sneppen model, the two dynamic exponents obtained starting from both a flat interface and a critical (saturated) state show the same behavior as our model [19].

In order to obtain the roughness exponent, we plot the saturated value of $W^2(L)$ versus system size L in double logarithmic scale. The obtained roughness exponent is $\alpha \simeq 1.15$ as shown in Fig. 3. From these numerical results, the dynamic exponent is obtained as $z = \alpha/\beta \simeq 1.29$. The roughness exponent value measured from the numerical integration of the QEW equation and automaton model in QEW universality class is about 1.0–1.25 [14–16,18]. Therefore the roughness exponent obtained from our model is consistent with the results for the QEW universality class.

We have also measured the height-height correlation function $C(x)$ defined as

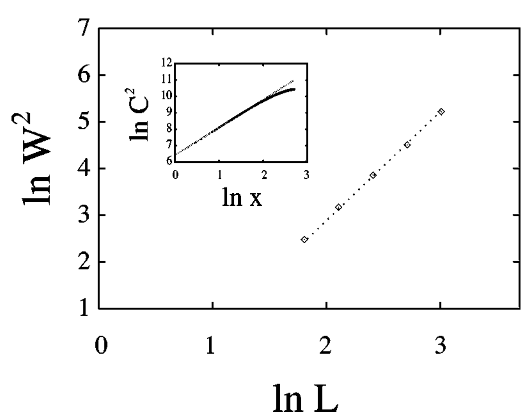

FIG. 3. Plot of $W^2(L,t)$ vs system size L in double logarithmic scales for the system sizes $L=64$, 128, 256, 512, and 1024. The line obtained from the least squares fits has the slope $2\alpha = 2.3$. Inset: Plot of $C^2(x)$ vs x in double logarithmic scales for the system size $L=1024$. The line obtained from the least squares fits has the slope $2\alpha_c = 1.7$.

$$C(x)=\left\langle \frac{1}{L^{d'}}\sum_{x}[h(x+x_1,\tau)-h(x_1,\tau)]^2\right\rangle^{1/2}, \qquad (4)$$

where time τ is larger than the saturation time, and $C(x)$ scales as x^{α_c}. The roughness exponent value from $C(x)$ is $\alpha_c \simeq 0.85$ as shown in the inset of Fig. 3. This value is smaller than the one obtained from the global interface width. It is known that this anomalous scaling of the local width is due to the super-roughening, in such a way that the roughness exponent α_c obtained from the height-height correlation function is smaller than the one obtained from the saturated value of $W^2(L)$ [14–16]. Super-rough scaling occurs when the roughness exponent of the global width is $\alpha > 1$. Super-rough interfaces do not represent the self-affine scaling nature since the basic step is a diverging quantity [14]. We have obtained an averaged basic step, $\langle |h(x+1,t) - h(x,t)|\rangle \sim L^{0.26}$, which diverges as $L \to \infty$. The height-height correlation function in the super-rough interfaces might be given as $C(x) \sim x^{\alpha_c} L^{\alpha-\alpha_c}$. If the correlation length x is the same as the system size L, we recover $C(x=L) = W(L)$. Roux and Hansen [14] and Galluccio and Zhang [20] obtained the scaling results for the roughness exponent $\alpha_c \simeq 0.85$ and $\alpha \simeq 1.20$ from the numerical integration of the QEW equation. These results are in agreement with our results.

Next, let us compare the properties of self-organized criticality between our model and the Sneppen model. The dynamics of the two models evolves through the process of the coherent activity after a transient period. The coherent activity means that new updatings are much more likely to occur on the newly updated sites. The number of the site having an updated random number on the surface per each time is one, two, or three in our model but the number is generally larger in the Sneppen model due to the instantaneous avalanches to satisfy the RSOS condition. Hence the area of active zone on the substrate is generally much larger in the Sneppen model than in our model. Figure 4 shows the snapshots of the growing interface at two configurations after saturation, supporting the above argument.

A well known method to classify the universality class of stochastic growth models with quenched noise is to measure the dependence of the velocity of the driven interface on the slope of a tilted substrate after saturation [21]. In the stochastic model belonging to the QKPZ universality class, the number of the particles added on the surface generally increases as the slope of the tilted substrate becomes larger because of the presence of the KPZ nonlinear term, so that the velocity of the driven interface depends on the slope of the tilted substrate. However, in the model belonging to the QEW universality class, the mean velocity does not change along with the increase of the slope of the tilted substrate. The velocity of our model is always $v=(1/L)\{\Sigma_x^L h(x,t+1) - \Sigma_x^L h(x,t)\} = 1/L$ regardless of the slope of the substrate tilt. It is because only one particle in our model is added on the interface per each time. We thus argue that our model belongs to the QEW universality class.

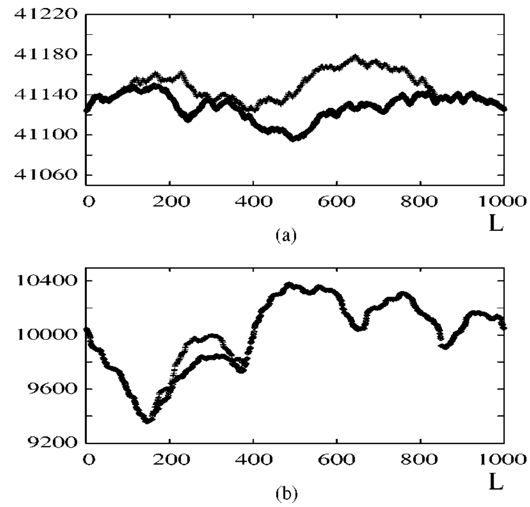

FIG. 4. Snapshots of the Sneppen model (a) and our model (b) for the system size $L=1024$ and at different times. The area of both active zones is the same.

In summary, we have introduced a simple self-organized stochastic growth model which belongs to the QEW universality class. The obtained roughness and growth exponents are $\alpha \simeq 1.15$, $\beta \simeq 0.89$ (before saturation) and 0.75 (after saturation), respectively. These exponent values coincide well with those from the discretized models in the QEW universality class and direct integrations of the QEW equation. There is no slope dependence of interface velocity in our model, indicating the generic behavior for the QEW universality class.

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